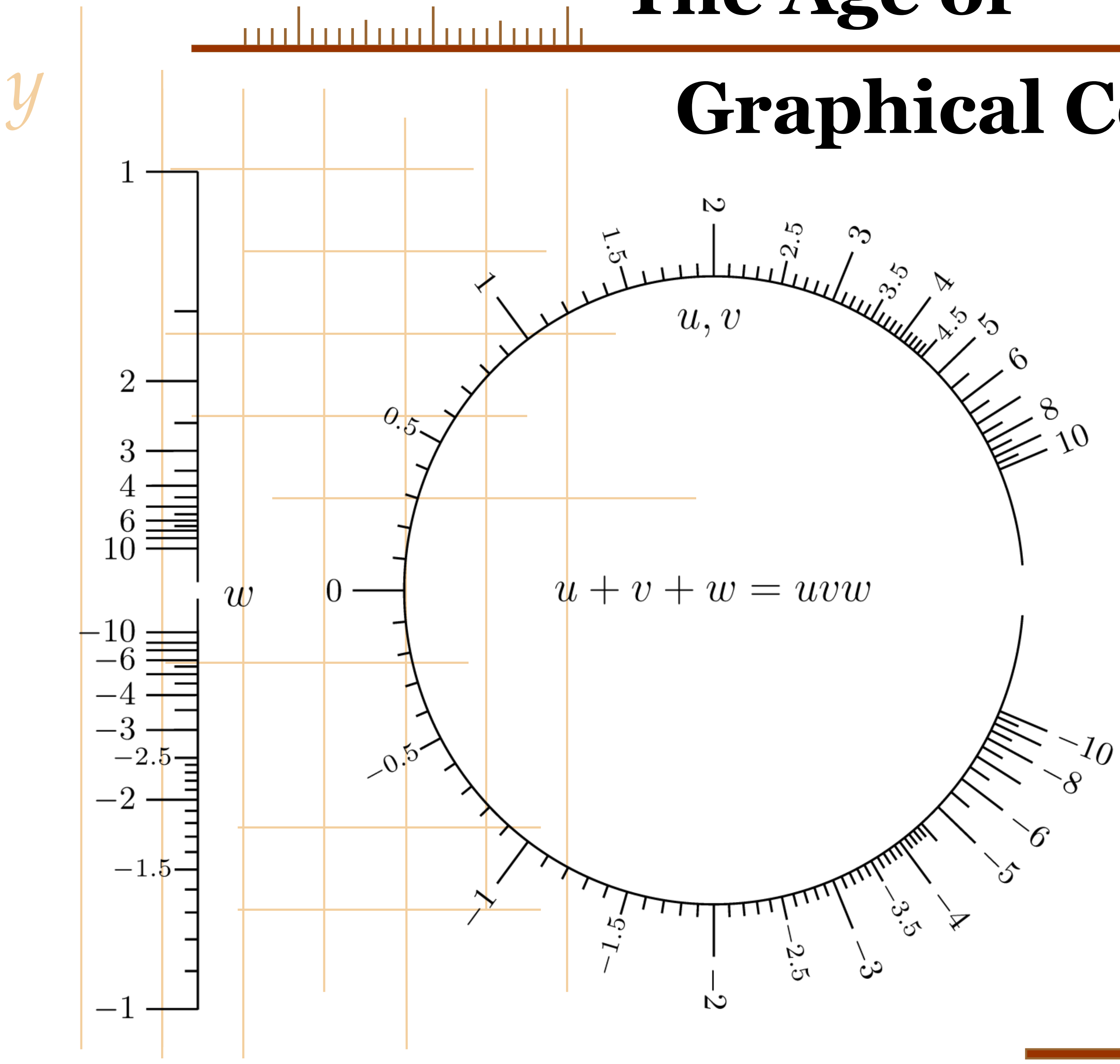
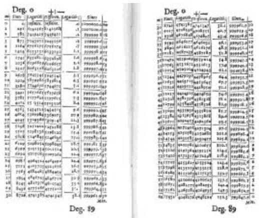
The Age of
Graphical Computing

A.D. 1844
A.D. 1974



A 2010
Calendar

Introduction



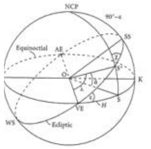
It is difficult for us today to grasp the drudgery of complex arithmetic calculations, or even repeated simpler calculations, in the past. This was especially true with repetitive computations that required tables of roots, logarithms and trigonometric functions in such fields as astronomy, navigation, surveying, and a wide variety of military and engineering applications.

Have you ever had to calculate the positions of astronomical objects? Orbital calculations relative to an observer on the Earth require derivations and time-consuming solutions of spherical trigonometric equations. And yet these kinds of calculations were accomplished by ancients such as Vitruvius and Ptolemy in the days prior to the advent of calculators or computers, or even trigonometry or algebra, using methods of Descriptive Geometry that are rarely taught today.

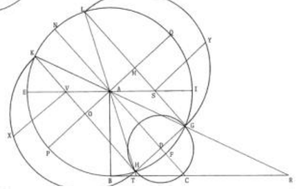
$$z_H = \arccos\left(\sin\phi \sin\delta + \cos\phi \cos\delta \cos\tau\right)$$

$$A_H = \arccos\left(\frac{\sin\delta\cos\phi - \sin\phi\cos\delta\cos\tau}{\sin z_H}\right)$$

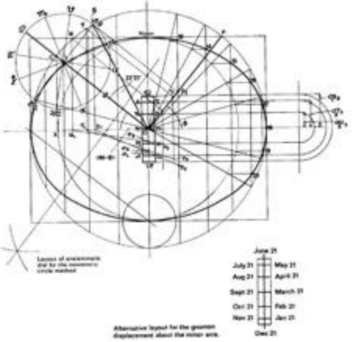
where: ϕ = terrestial latitude
δ = current solar declination
$\tau = (n/6)\arccos\left(-\tan\phi\tan\delta\right)$
n = the number of unequal hours before or after local noon
z_H = the zenith angle of the sun
A_H = the altitude of the sun above the horizon

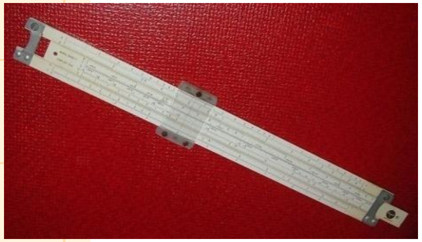


The Greeks folded (*rabatted*) the fundamental great circles onto the page and performed intricate geometrical constructions to map the Earth-Sun relative motion and incorporate local measurements into global maps and sophisticated sundials.

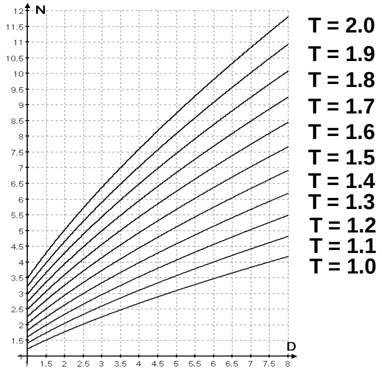


Astrolabes, quadrants and other volvelles and dials evolved to perform more complex computations in graphical form. In 1610-1614, Joost Bürgi and John Napier invented logarithms, and mathematicians and scientists such as Johann Kepler created tables of logarithms to aid in computation. William Oughtred and others developed the slide rule in the 1600s based on the properties of logarithms, and the slide rule continued its dominant role in non-graphical computation until the early 1970s. The slide rule provided the greatest versatility in computing the vast variety of equations, but it required multiple error-prone steps to provide solutions, effort that was not decreased even when solving one equation repetitively.

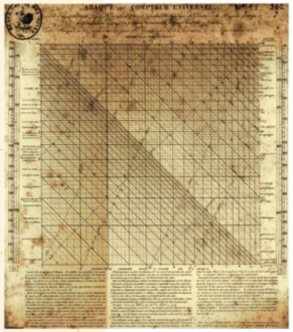




Meanwhile, on the graphical front Rene Descartes created the Cartesian coordinate system in the 17th century, and mathematicians over the next two centuries laid the foundation for applied numerical mathematics in large part on this field of analytical geometry. A two-dimensional graph provided fast solutions to an engineering precision for a single equation in two variables, and more complicated families of curves or so-called *intersection charts* extended the use of Cartesian graphs to one additional variable.



Introduction



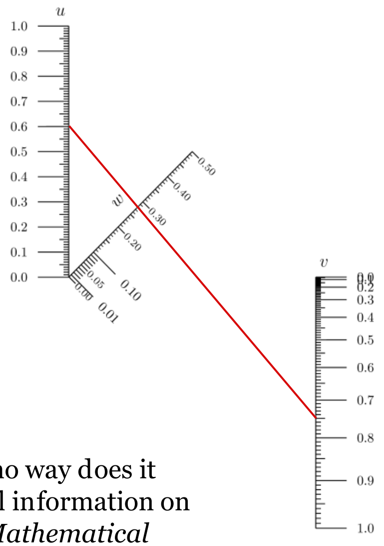
In 1844 Leon Lalanne succeeded in linearizing the curves $y=x^p$ by plotting the first log-log plot in history, thereby creating his *Universal Calculator*, chock-full of lines for common engineering calculations and capable of graphically computing formulas in powers or roots of *x* (or of trigonometric functions in *x)* with ease. The year 1844 is taken here as the start of the *Age of Graphical Computing*. Other graphical methods evolved, and ultimately the field of nomography was invented in 1880 by Maurice d'Ocagne, a breakthrough in graphical computing so radical that it dominated the field of graphical computing until the spread of computers and electronic calculators in the early 1970s.

This 2010 calendar predominantly treats the field of nomography and the amazing variety of nomograms that can be created from it. A nomogram is a layout of graphical scales for computing formulas of 3 or more variables using a straightedge such as a ruler or the edge of a sheet of paper. A drawn or imagined *isopleth* connects matching values of variables for a particular formula, so if all variables but one is known, the unknown variable can be read off the intersection of the isopleth with its scale. Variables that cannot be isolated algebraically can be read directly off a nomogram. Beyond their practical use, the scales of a nomogram often create geometric figures and curves of a certain beauty and flair, influenced to a striking degree by the cleverness of the nomographer. Simple nomograms can be seen today at times in engineering catalogs and medical offices, but the really creative ones, the ones that universally draw interest and display the wondrous virtuosity of mathematics, are nowhere to be found anymore.

Nomograms can be created with geometric relations, but the more extraordinary ones are nearly always created using a method of determinants developed by d'Ocagne. Sometimes in this calendar you will see an equation adjacent to a nomogram, in which the determinant of a matrix is set equal to zero. When the determinant is expanded, you will see that the resulting equation matches the overall equation of the nomogram. If the determinant is in a form where no variable appears in more than one row and the last column is all 1's, then the first two elements in each row represent the (x,y) location of the scale point for values of the variable(s) in that row. For example, using the rules for expanding a determinant the equation $w= u/(u + v^2 + 1)$ or $uw + v^2w + w - u = 0$ can be expressed as

$$\begin{vmatrix} 0 & u & 1 \\ 1 & -v^2 & 1 \\ w & w & 1 \end{vmatrix} = 0$$

so a tick on the u-scale lies at (0,u) for every u, (or in other words the u-scale is a linear, vertical scale), the ticks on the v-scale are at $(1,-v^2)$, and the ticks on the w-scale are at (w,w) resulting in a linear 45 degree scale.



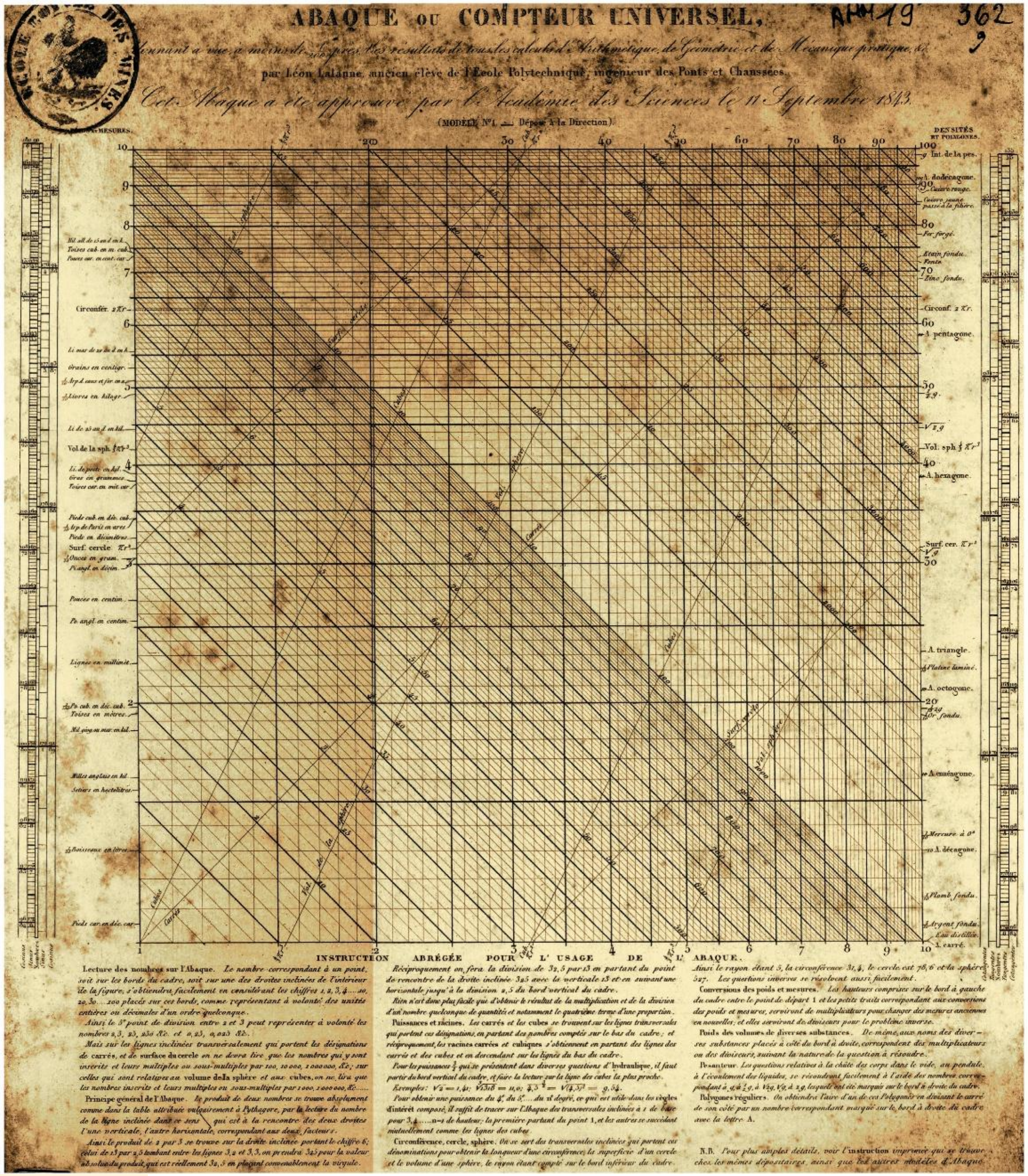
The nomograms in this calendar are representative of some of the variety once in use for graphical computing, but in no way does it approach a significant survey of this rich field of study. Perhaps a 2011 calendar will consider other designs. Additional information on nomograms and other topics in this calendar can be found in articles on my blog, "*Dead Reckonings: Lost Art in the Mathematical Sciences*" at http://www.myreckonings.com/wordpress. I hope you have a happy year in 2010.

Ron Doerfler

rondoerfler@myreckonings.com

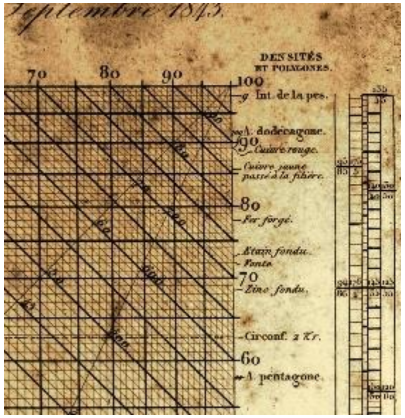
Most of the nomograms herein were created with the PyNomo software package of Leif Roschier found at http://www.pynomo.org. The calendar pages are based on an InDesign template created by Juliana Halvorson at http://www.graphmaster.com/calendarinstructions/.

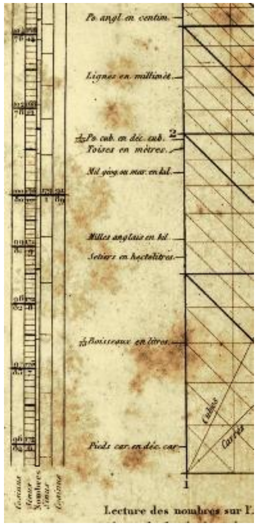
Lalanne's Universal Calculator

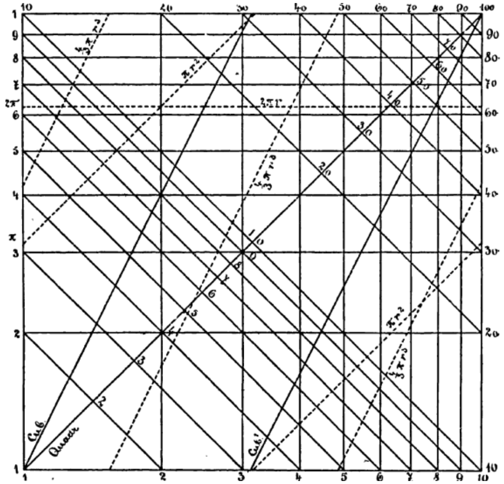


In 1844, Leon LaLanne created the first log-log plot in history, his *Universal Calculator*.

The product of x and y is found from their intersection with the 45° lines, squares at the 45° line from the origin, cubes from the steeper (*Cub*) line from the origin or its wraparound, and various engineering and chemical formulas of roots and powers at their lines. Following the line to the edge continues a calculation to additional terms.







Trigonometric functions are plotted along the sides for use (or use of their inverses) in calculations as well.

LaLanne envisioned copies of his *Universal Calculator* posted in public squares and business meeting places for popular use.

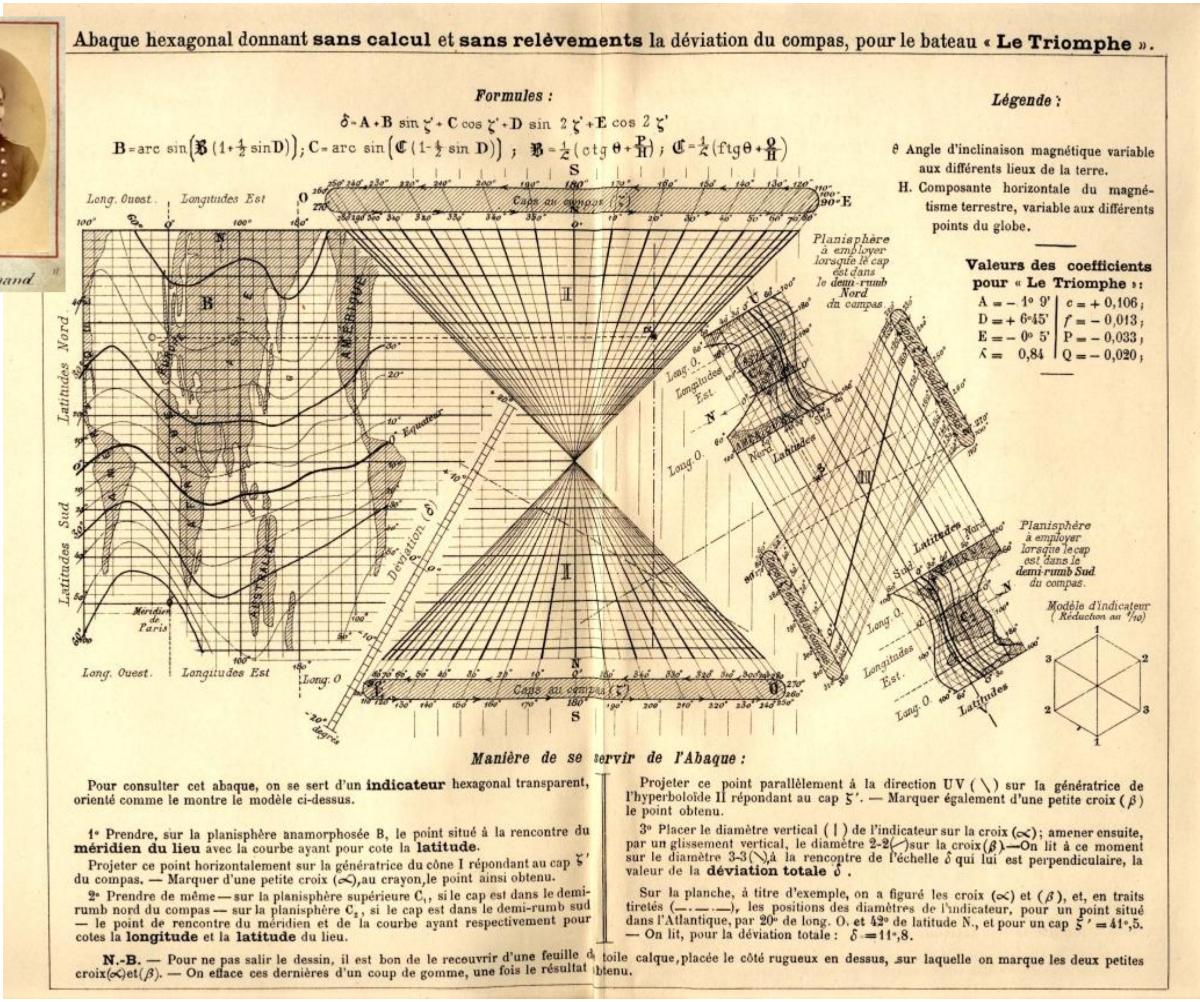
JANUARY

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
December 2009 S M T W T F S 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	February 2010 S M T W T F S 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28				1 **New Year's Day**	2
3	4	5	6	7	8	9
10	11	12	13	14	● 15	16
17	18 **Martin Luther King Day**	19	20	21	22	23
24	25	26	27	28	29	○ 30
31						

Lallemand's *L'Abaque Triomphe*

In 1885, Charles Lallemand, director general of the geodetic measurement of altitudes throughout France, published a *hexagonal chart* for determining the compass course correction (the magnetic deviation) due to iron in the ship, *Le Triomphe* for any navigable location on Earth. It is a stunning piece of work, combining measured values of magnetic variation around the world with eight magnetic parameters of the ship also measured experimentally, all merged into a very complicated formula for magnetic deviation as seen at the top of the chart.


Lallemand


Abaque hexagonal donnant **sans calcul** et **sans relèvements** la déviation du compas, pour le bateau « **Le Triomphe** ».

Formules :

$$\delta = A + B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta' + E \cos 2\zeta'$$

$$B = \arc \sin\left(\mathfrak{B}\left(1+\tfrac{1}{2}\sin D\right)\right);\ C = \arc \sin\left(\mathfrak{C}\left(1-\tfrac{1}{2}\sin D\right)\right);\ \mathfrak{B} = \tfrac{1}{\lambda}\left(c\,tg\,\theta + \tfrac{P}{H}\right);\ \mathfrak{C} = \tfrac{1}{\lambda}\left(f\,tg\,\theta + \tfrac{Q}{H}\right)$$

Légende :

θ Angle d'inclinaison magnétique variable aux différents lieux de la terre.

H. Composante horizontale du magnétisme terrestre, variable aux différents points du globe.

Valeurs des coefficients pour « Le Triomphe » :

A = − 1° 9'	c = + 0,106 ;
D = + 6°45'	f = − 0,013 ;
E = − 0° 5'	P = − 0,033 ;
λ = 0,84	Q = − 0,020 ;

Manière de se servir de l'Abaque :

Pour consulter cet abaque, on se sert d'un **indicateur** hexagonal transparent, orienté comme le montre le modèle ci-dessus.

1° Prendre, sur la planisphère anamorphosée B, le point situé à la rencontre du **méridien du lieu** avec la courbe ayant pour cote la **latitude**.

Projeter ce point horizontalement sur la génératrice du cône I répondant au cap ζ' du compas. — Marquer d'une petite croix (∝), au crayon, le point ainsi obtenu.

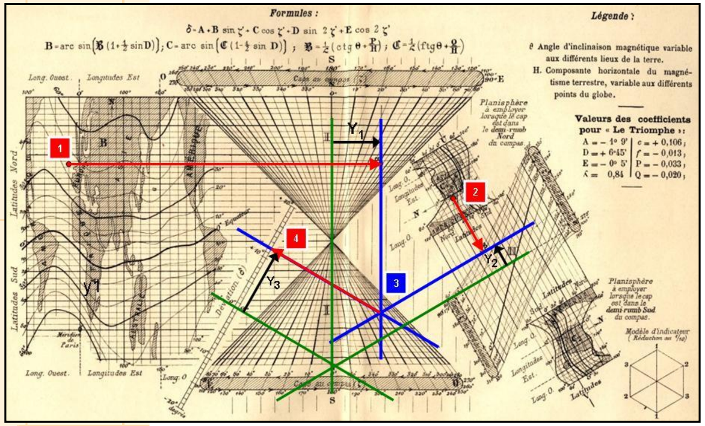
2° Prendre de même — sur la planisphère supérieure C_1, si le cap est dans le demi-rumb nord du compas — sur la planisphère C_2, si le cap est dans le demi-rumb sud — le point de rencontre du méridien et de la courbe ayant respectivement pour cotes la **longitude** et la **latitude** du lieu.

Projeter ce point parallèlement à la direction UV (\ ) sur la génératrice de l'hyperboloïde II répondant au cap ζ'. — Marquer également d'une petite croix (β) le point obtenu.

3° Placer le diamètre vertical (|) de l'indicateur sur la croix (∝) ; amener ensuite, par un glissement vertical, le diamètre 2-2 (/) sur la croix (β). — On lit à ce moment sur le diamètre 3-3 (\ ), à la rencontre de l'échelle δ qui lui est perpendiculaire, la valeur de la **déviation totale** δ .

Sur la planche, à titre d'exemple, on a figuré les croix (∝) et (β), et, en traits tiretés (—.—.—), les positions des diamètres de l'indicateur, pour un point situé dans l'Atlantique, par 20° de long. O. et 42° de latitude N., et pour un cap ζ' = 41°,5. — On lit, pour la déviation totale : δ = 11°,8.

N.-B. — Pour ne pas salir le dessin, il est bon de le recouvrir d'une feuille de toile calque, placée le côté rugueux en dessus, sur laquelle on marque les deux petites croix (∝) et (β). — On efface ces dernières d'un coup de gomme, une fois le résultat obtenu.



The sample calculation on the chart is described above:

1. The ship latitude and longitude is located among the curved lines on the leftmost map and a horizontal line is extended to the compass course ζ' in the center hourglass grid (a distance Y_1 = B sin ζ' from the vertical green line)
2. The ship latitude and longitude is found in the upper (for a northerly heading) or lower (for a southerly heading) map and a line parallel to the grid is extended to the corresponding compass course in the twisted grid (a distance Y_2 = A + C cos ζ' + D sin 2 ζ' + E cos ζ' from the angled green line).
3. A translucent hexagonal overlay (shown in blue) is overlaid so that two arms pass through the two marked points. Through a geometric exercise, it can be shown that the magnetic deviation (*compass correction*) Y_3 on the third scale is the sum of Y_1 and Y_2.

ζ' is the current ship compass reading (or *compass course*)

H and θ are the horizontal component and dip angle of the Earth's magnetic variation at the ship location

A, D, E, λ, c, f, P and Q are magnetic parameters measured for *Le Triomphe*

FEBRUARY

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	1	2	3	4	5	6
7	8	9	10	11	12	● 13
14 **Valentine's Day**	15 **President's Day**	16	17	18	19	20
21	22	23	24	25	26	27
○ 28						

January 2010

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

March 2010

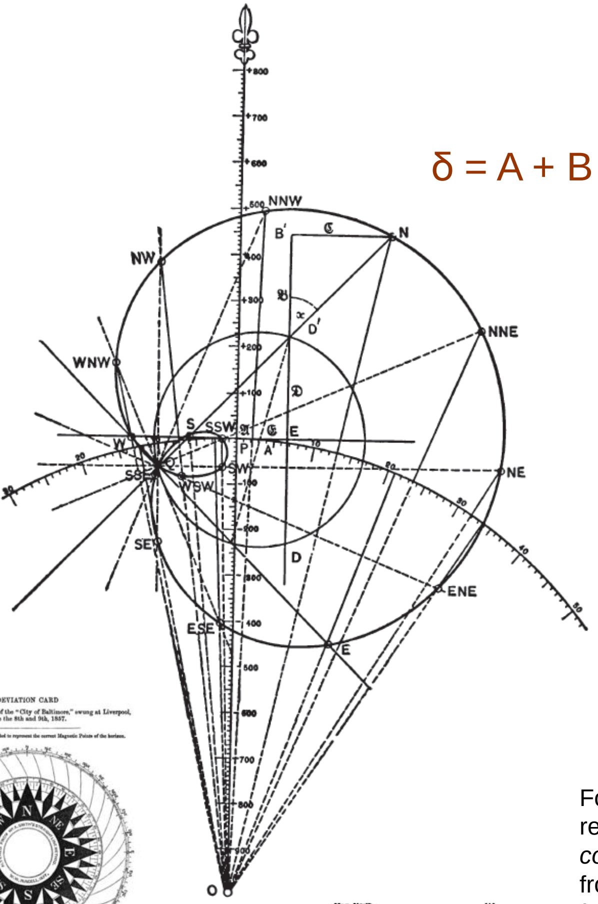
S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

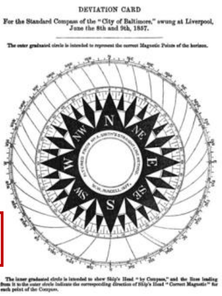
Dygograms

The Scottish mathematician and lawyer Archibald Smith first published in 1843 his equations for the magnetic deviation of a ship, or in other words, the error in the ship's compasses from permanent and Earth-induced magnetic fields in the iron of the ship itself. This effect had been noticed in mostly wooden ships for centuries, and broad attempts to minimize it were implemented. But the advent of ships with iron hulls and steam engines in the early 1800s created a real crisis. A mathematical formulation of the deviation for all compass courses for a location at sea was needed in order to understand and compensate for it. Smith invented graphical methods for quickly calculating the magnetic deviation for any ship's course once ship parameters were found, geometric constructions called dynamogoniograms (force-angle diagrams), or *dygograms* for short.

$$\delta = A + B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta' + E \cos 2\zeta'$$

δ is the magnetic deviation (*compass correction*)
ζ' is the ship compass reading (*compass course*)
A, D, E, λ, c, f, P and Q are magnetic parameters measured for the ship
H and θ are the horizontal component and dip angle of the Earth's magnetic variation at the ship location
A = arcsin ***A***
B = arcsin [***B*** / (1 + ½ sin D)]
C = arcsin [***C*** / (1 - ½ sin D)]
D = arcsin ***D***
E = arcsin ***E***
A, D, E = constants for ship
B = (1/λ) (c tan θ + P / H)
C = (1/λ) (f tan θ + Q / H)





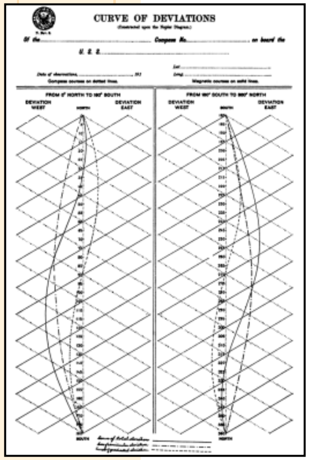
DEVIATION CARD

For the Standard Compass of the "City of Baltimore," swung at Liverpool, June the 8th and 9th, 1857.

The outer graduated circle is intended to represent the correct Magnetic Points of the horizon.

The inner graduated circle is intended to show Ship's Head "by Compass," and the lines leading from it to the outer circle indicate the corresponding direction of Ship's Head "Correct Magnetic" for each point of the Compass.

Alternate graphical computers



CURVE OF DEVIATIONS

(Constructed upon the Napier Diagram.)

Of the .... Compass No. .... on board the

U. S. S. ....

Date of observations, ..., 191

Lat. ....

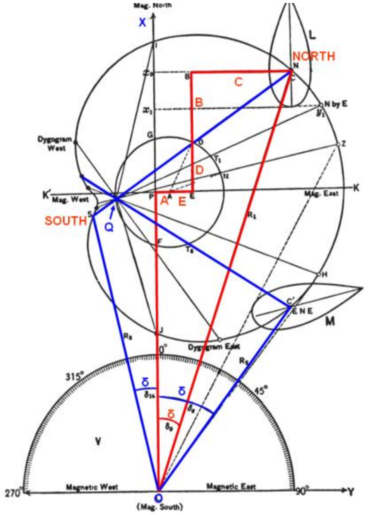
Long. ....

Compass courses on dotted lines.

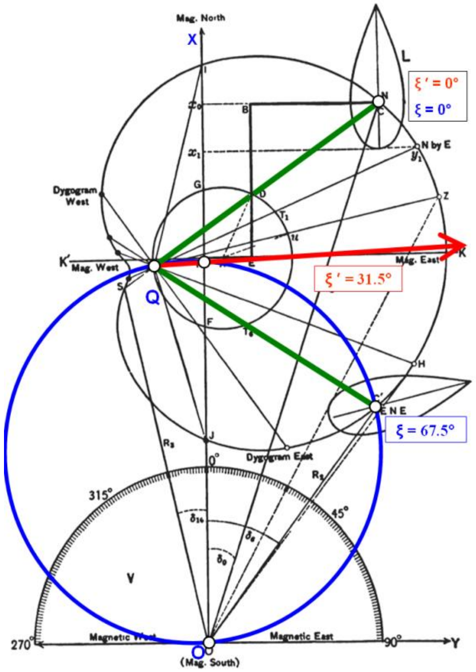
Magnetic courses on solid lines.

FROM 0° NORTH TO 180° SOUTH	FROM 180° SOUTH TO 360° NORTH
DEVIATION WEST · NORTH · DEVIATION EAST	DEVIATION WEST · SOUTH · DEVIATION EAST

To construct a dygogram, find the North (*N*) position by laying out from *O* the lengths ***A***, ***B***, ***C***, ***D*** and ***E*** as shown. Draw a circle centered at *A* and passing through *D*. The magnetic deviation δ for a *magnetic course* ζ of North (0) is the angle XON read on the protractor.
Now extend ND the same distance beyond *D* to find the South (*S*) point. The point *Q* is the intersection with the circle. Continue to create the *Limaçon of Pascal* figure by moving the midpoint of the segment NS along the circle and marking the endpoints.



For any ship compass reading (the *compass course ζ'*), draw a line from Q at this angle from QN (the red arrow here) and mark the point where it crosses the vertical line OX. Then with dividers construct an arc that passes through O, Q, and this point (the blue circle) and mark a new point where it crosses the limaçon (the *magnetic course ζ*). The magnetic deviation δ is the angle between OX and this new point as read on the protractor.



MARCH

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	● 15	16	17	18	19	20
21	22	23	24	25	26	27
28	○ 29	30	31		February 2010 S M T W T F S 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28	April 2010 S M T W T F S 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

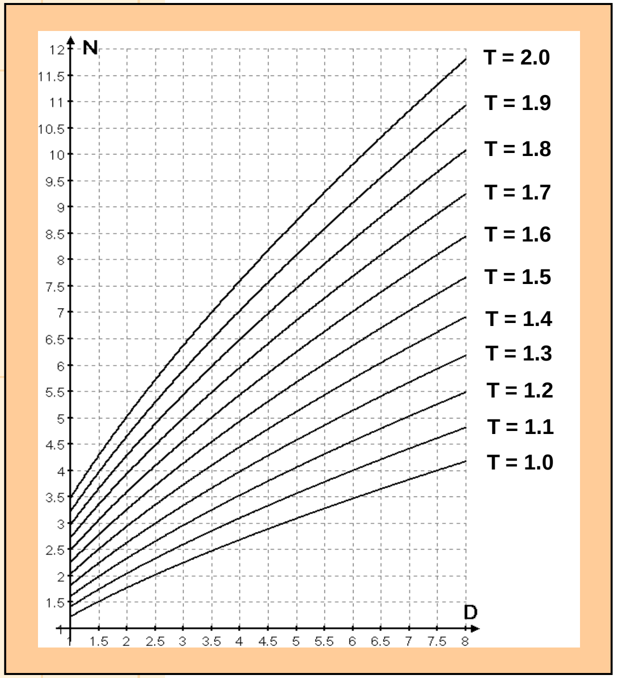
Nomography

Nomograms solve equations in 3 or more variables, providing lightning fast, easy calculations to an engineering precision in a form that is easy to reproduce on a photocopier

Maurice d'Ocagne in his student days at the École Polytechnique. (Photograph courtesy of the Bibliothèque Centrale, École Polytechnique.)

Nomography was invented in 1880 by Maurice d'Ocagne and was used extensively for many years to provide engineers with fast graphical calculations of complicated formulas to a practical precision.

A traditional three-variable graph



Much simpler to plot

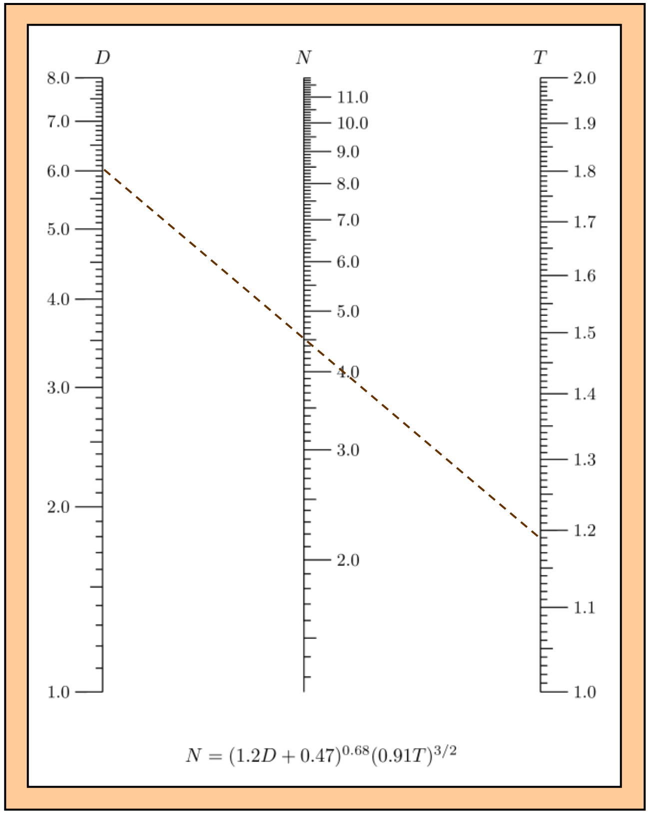
No family of curves or grid

Much finer resolution

Less prone to mistakes

Can be extended to additional variables

Example: $N = (1.2D + 0.47)^{0.68}(0.91T)^{3/2}$



A parallel-scale nomogram

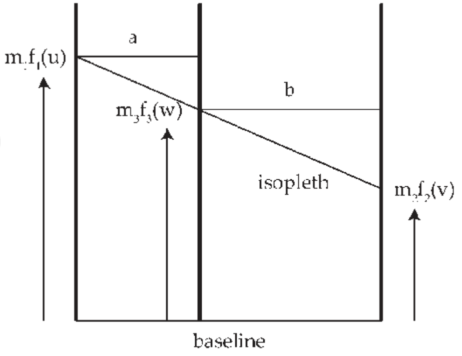
A straightedge (such as the edge of a sheet of paper or a string) called an *isopleth* is used to connect known values to find the unknown value.

The simplicity of a nomogram can be startling!

Parallel-Scale Design: $f_1(u) + f_2(v) = f_3(w)$

Take logarithms to convert the equation to a sum

$$0.68 \log(1.2D + 0.47) + 1.5 \log T = \log N – 1.5 \log 0.91$$



m_1 = height/range for D scale

m_2 = height/range for T scale

$m_3 = m_1m_2/(m_1+m_2)$

$a/b = m_1/m_2$ and $a+b$ = width

APRIL

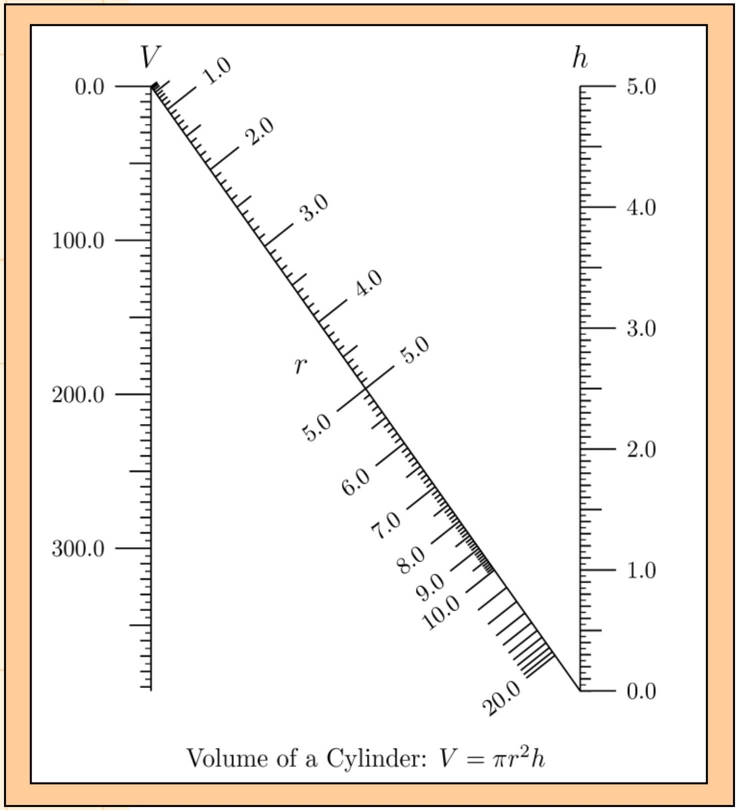
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
March 2010 S M T W T F S 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	May 2010 S M T W T F S 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31			1	2	3
4 **Easter Sunday**	5	6	7	8	9	10
11	12	13	● 14	15	16	17
18	19	20	21	22	23	24
25	26	27	○ 28	29	30	

Two Classic Nomogram Designs

Division

$$f_3(w) = \frac{f_1(u)}{f_2(v)}$$

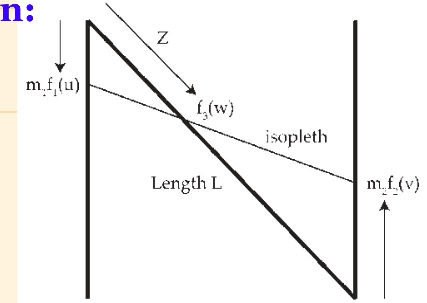
Here we have $r^2 = V/\pi h$



Volume of a Cylinder: $V = \pi r^2 h$

An "N" or "Z" Chart

Design:



$$Z = \frac{L f_3(w)}{(m_2/m_1) + f_3(w)}$$

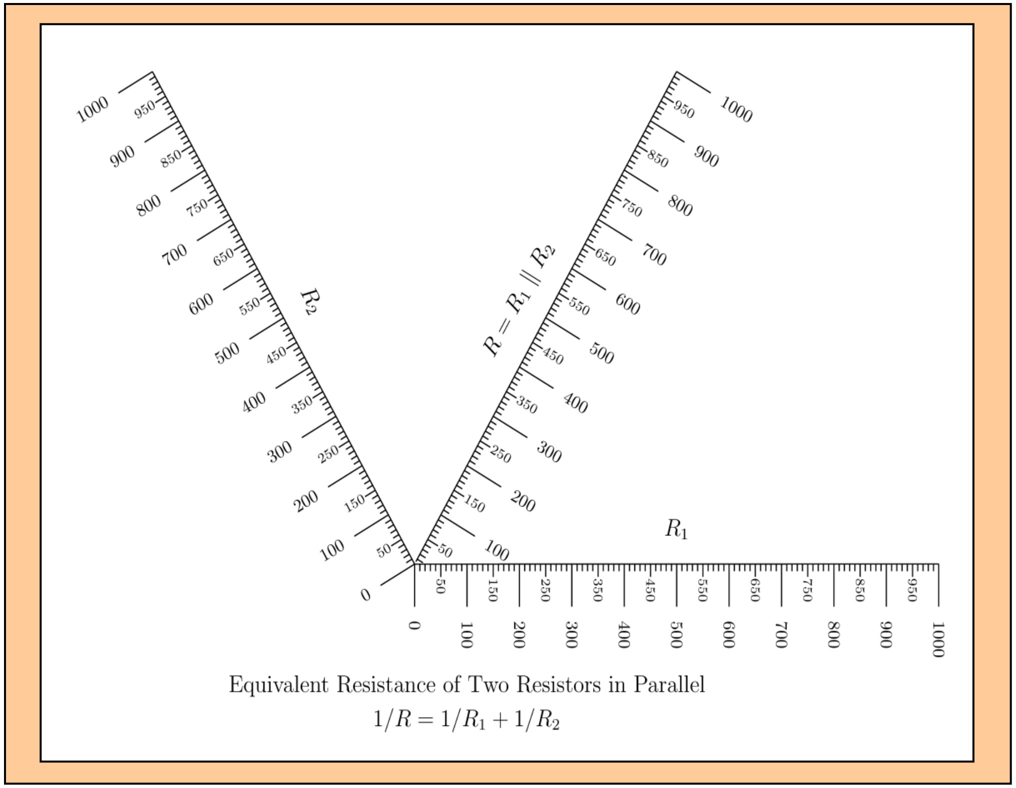
The diagonal scale can be floating segment, thus appearing "rather more spectacular" to the casual observer [Douglass 1947].

Harmonic Relation

$$\frac{1}{f_1(u)} + \frac{1}{f_2(v)} = \frac{1}{f_3(w)}$$

Design: $$m_1 = m_2 = \frac{m_3}{2\cos A}$$

where A is the angle between each of the 3 scales.
If A = 60° as below, then $m_1 = m_2 = m_3$.



Equivalent Resistance of Two Resistors in Parallel
$1/R = 1/R_1 + 1/R_2$

A Concurrent-Scale Nomogram

Standard resistor values can be marked so a convenient combination can be found by playing with the straightedge.

MAY

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
						1
2	3	4	5	6	7	8
9 **Mother's Day**	10	11	12	● 13	14	15
16	17	18	19	20	21	22
23	24	25	26	○ 27	28	29
30	31 **Memorial Day**					

April 2010

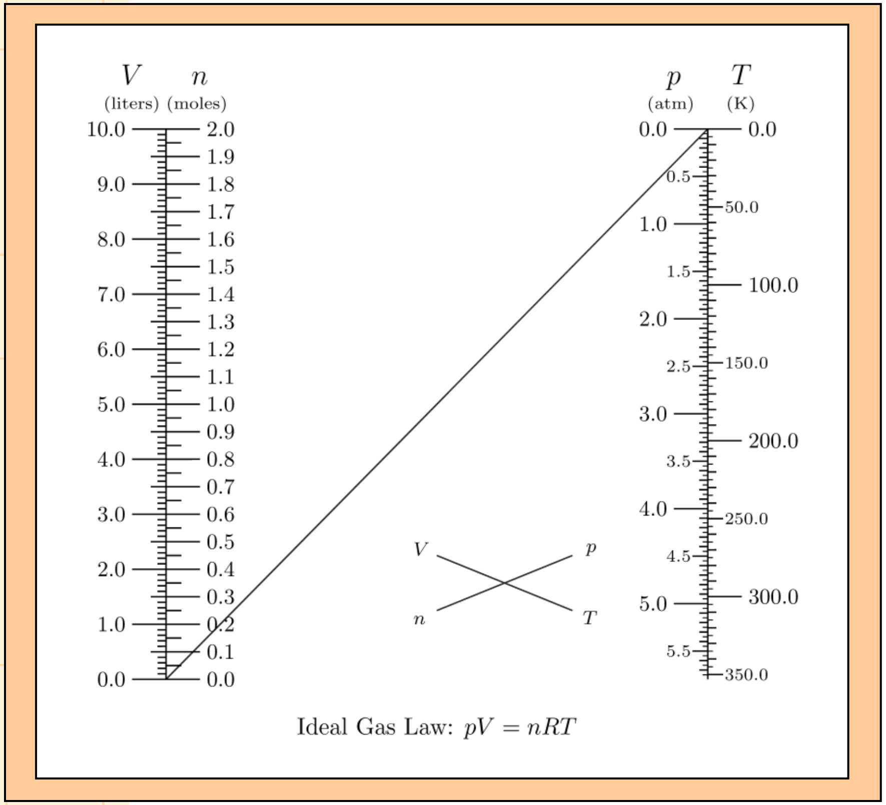
S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

June 2010

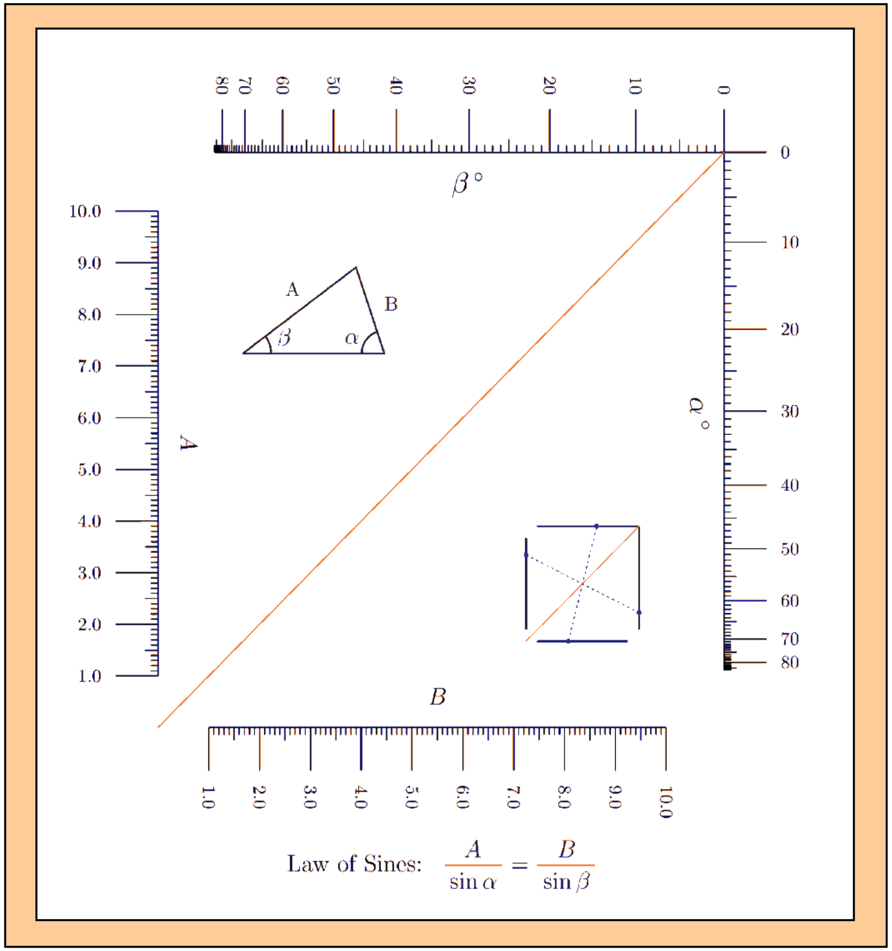
S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

Proportional Nomograms

4 Variable Proportion $\frac{f_1(u)}{f_2(v)} = \frac{f_3(w)}{f_4(t)}$

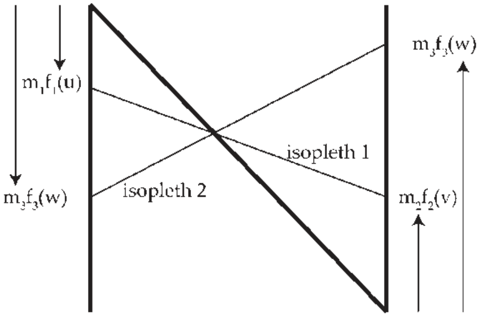


Ideal Gas Law: $pV = nRT$



Law of Sines: $\frac{A}{\sin\alpha} = \frac{B}{\sin\beta}$

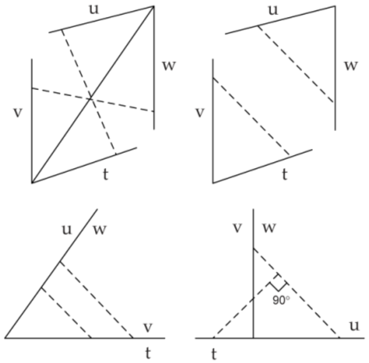
Proportional Design:



$$\frac{m_1}{m_2} = \frac{m_3}{m_4}$$

True for all types shown here

Other Layouts



JUNE

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6	7	8	9	10	11	● 12
13	14	15	16	17	18	19
20 **Father's Day**	21	22	23	24	25	○ 26
27	28	29	30		May 2010	July 2010

May 2010

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

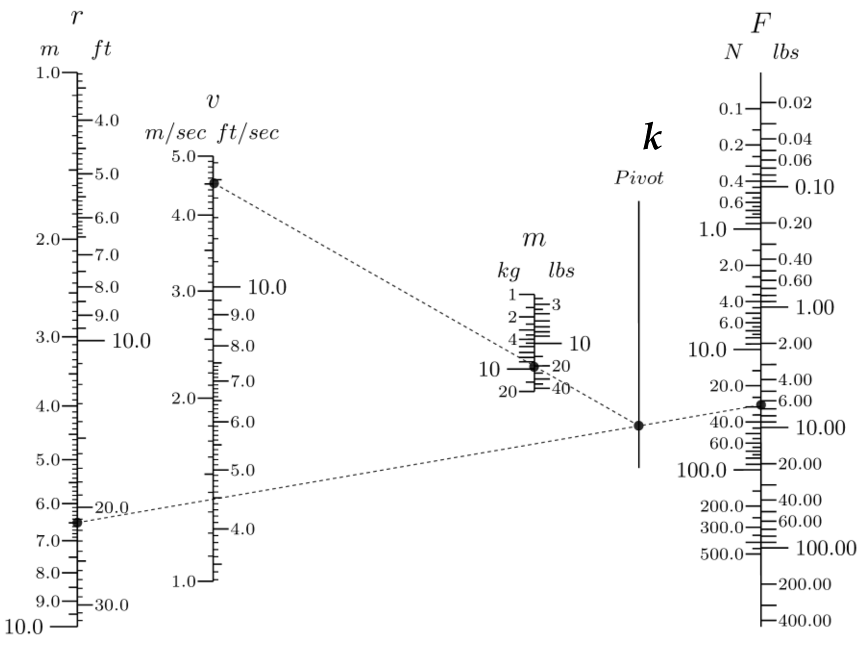
July 2010

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Compound Nomograms

Equations of more than three variables can be graphically computed using compound nomograms sharing scales.

The middle solution scale of the concurrent nomogram for two resistors in parallel can be used as the outer scale of a second nomogram to extend the nomogram for three resistors. A fourth parallel resistor can be added by seesawing back through the first set of scales, and so forth. A series resistor simply slides upward along the scale.



Centripetal Force: $F = mv^2/r$

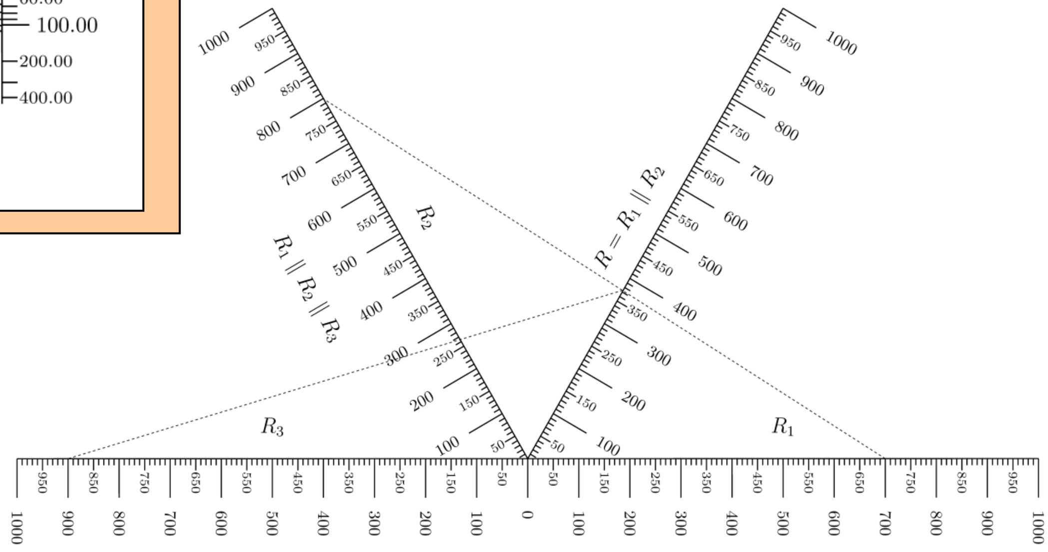
Compound Linear Design:

$$f_1(u) + f_2(v) = f_4(t) - f_3(w)$$

$$f_1(u) + f_2(v) = k$$

$$\rightarrow f_4(t) - f_3(w) = k$$

The k-scale is not labeled with scale values. It is called a *pivot line*.



Equivalent Resistance of Three Resistors in Parallel

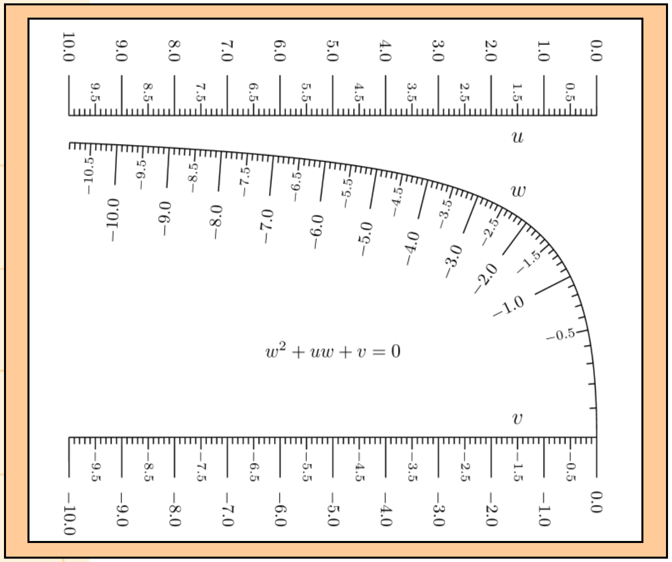
$1/R = 1/R_1 + 1/R_2 + 1/R_3$

Since the angle A between the scales is 60 , the scales are identical.

JULY

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
June 2010 S M T W T F S 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30	August 2010 S M T W T F S 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31			1	2	3
4 **Independence Day**	5 **Independence Day (Obs.)**	6	7	8	9	10
● 11	12	13	14	15	16	17
18	19	20	21	22	23	24
○ 25	26	27	28	29	30	31

Solving Polynomial Equations

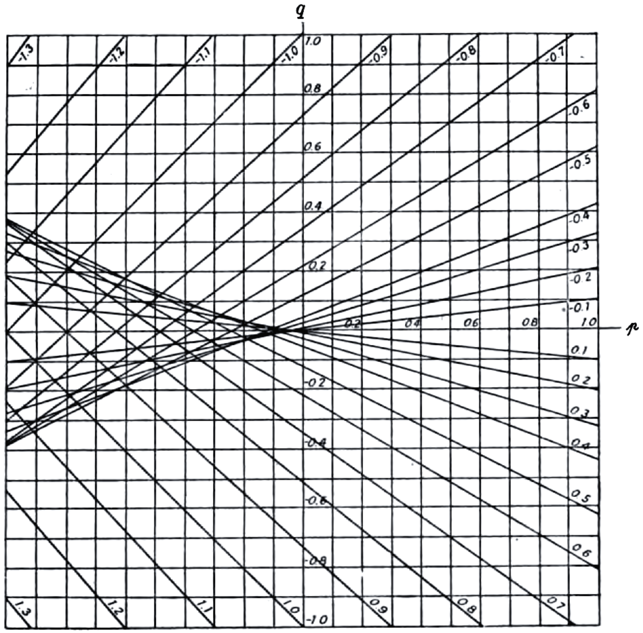


$$w^2 + uw + v = 0$$

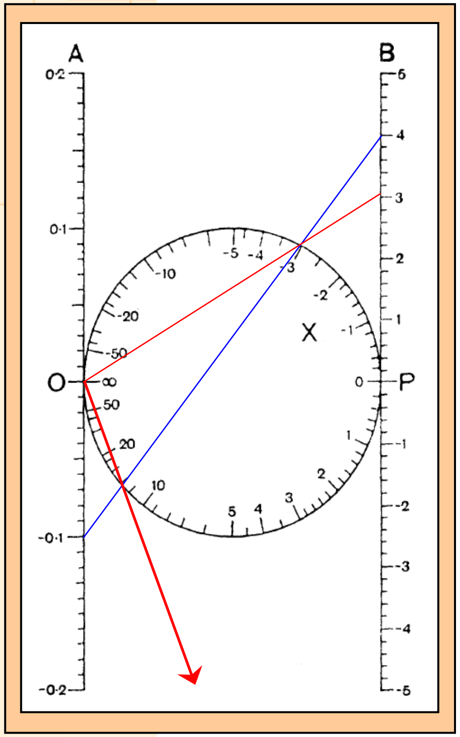
For a real root w_1 found here, the second root = $u+w_1$

$$\begin{vmatrix} -u & 1 & 1 \\ v & 0 & 1 \\ \frac{w^2}{w-1} & \frac{w}{w-1} & 1 \end{vmatrix} = 0$$

Finding real roots graphically



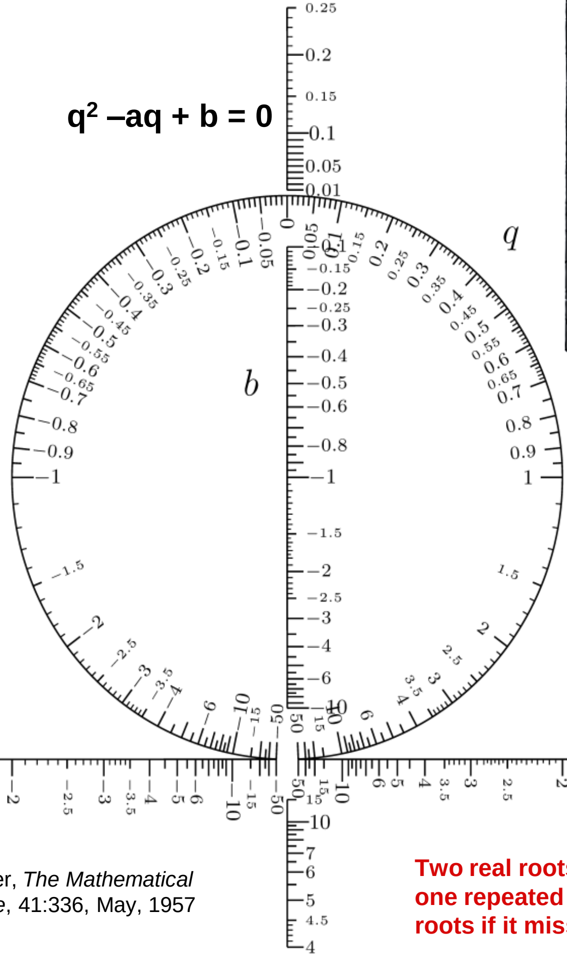
Not a nomogram, but a *network* or *intersection chart* for finding roots of the cubic equation $z^3 + pz + q = 0$. There will be one real root, 3 real roots of which two are equal, or 3 real roots, depending on whether z (interpolated between the slanted lines) lies outside of the triangular region, on its boundary, or within it. For example, p=0.6 and q=-0.4 gives z=0.47, while p=-0.8 and q=0.11 gives z=-0.96, 0.82, 0.14.



Linear scales and easy custom ranges of A and B for $Ax^2 + x + B = 0$

No need to graduate the circle! Read the roots as –B on lines from O through the marked location on the circle (here -3.06 for A=-0.1, B=4)

$$q^2 - aq + b = 0$$



Wheeler, *The Mathematical Gazette*, 41:336, May, 1957

Two real roots are found if the isopleth cuts the circle, one repeated real root if it touches the circle, no real roots if it misses the circle entirely

AUGUST

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2	3	4	5	6	7
8	● 9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	○ 24	25	26	27	28
29	30	31				

July 2010

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

September 2010

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

Astronomy

Once invented, nomograms were soon applied to time-consuming and repetitive calculations in celestial mechanics

Celestial Parallax: the difference between topocentric and geocentric location when observing comets and minor planets. Done with parallax correction, generally to two digits and in great number to define the orbits:

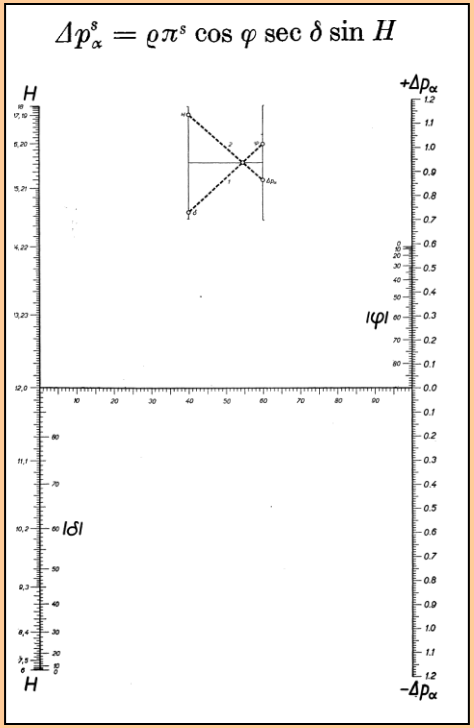
Δp_α = parallax factor

π^s = mean equatorial horizontal parallax of the sun in seconds

ρ = Earth radius to observation point in term of equatorial radius

φ = geocentric latitude of observer

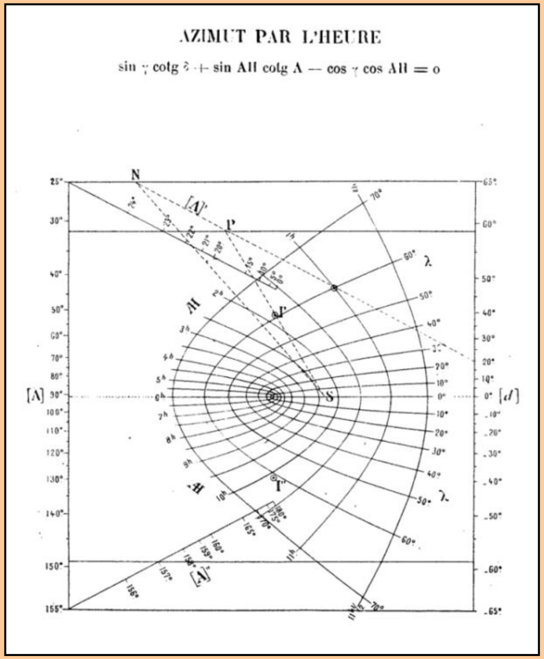
δ,H = declination and hour angle of body

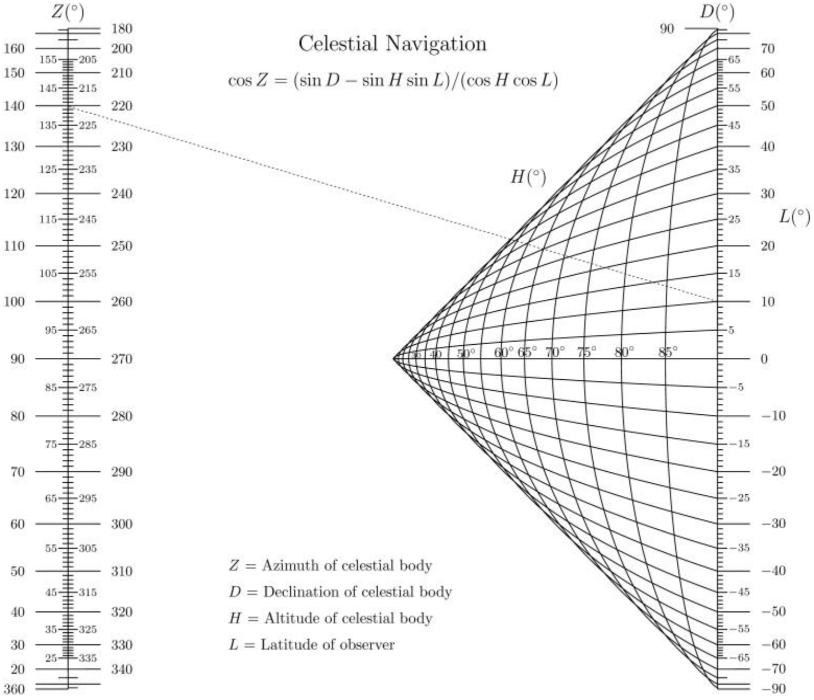


Kresàk, Bulletin of the Astronomical Institute of Czechoslovakia, 1957

Spherical Triangle relation between declination, latitude, altitude and azimuth

Spherical Triangle relation between declination, latitude, hour angle and azimuth

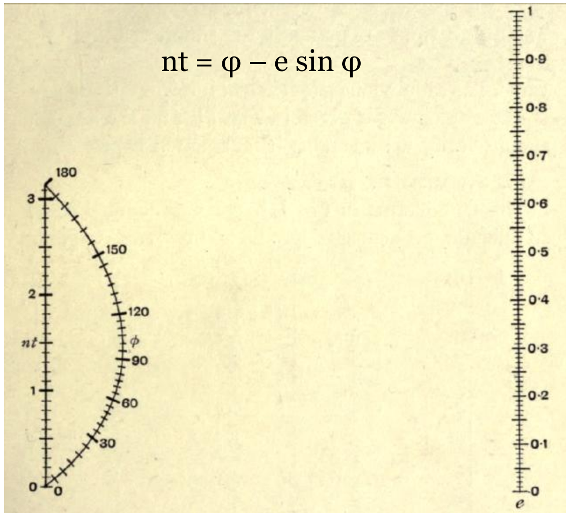




after Leif Roschier—see http://www.pynomo.org/wiki/index.php/Example:Star_navigation

Kepler's Equation for the relation between the polar angle φ of a celestial body in an eccentric orbit and the time elapsed from an initial point

This is an example of a nomogram solving for a variable (φ) that cannot be isolated algebraically.



SEPTEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2	3	4
5	6 **Labor Day**	7	● 8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	○ 23	24	25
26	27	28	29	30		

August 2010

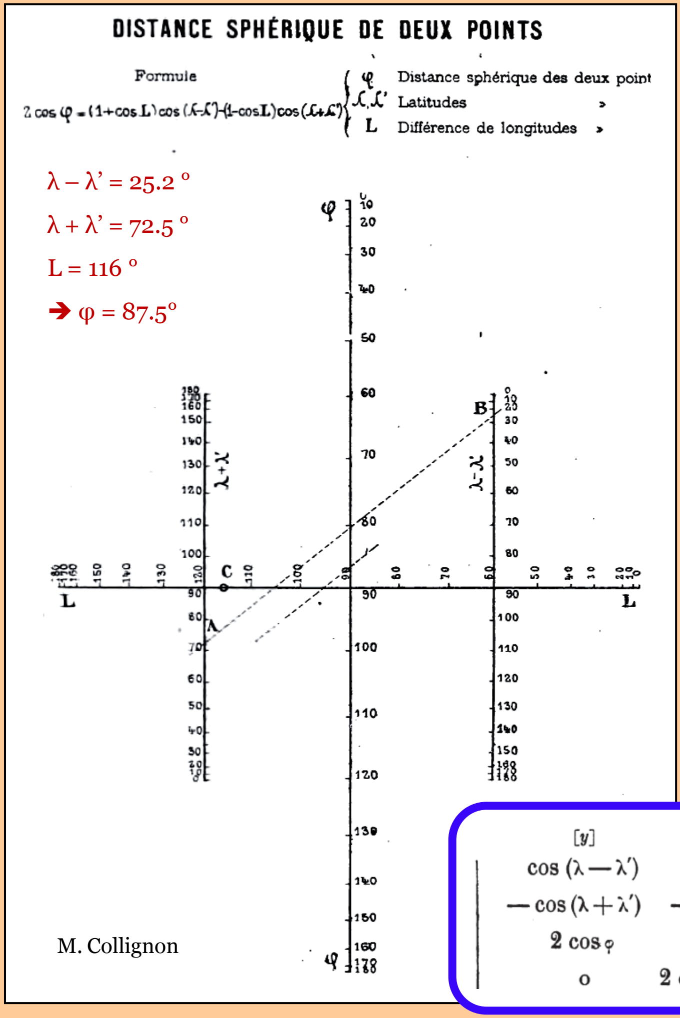
S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

October 2010

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

Navigation and Surveying

A 3D (4x4 determinant) nomogram solution for Great Circle Distance



λ – λ' = 25.2 °

λ + λ' = 72.5 °

L = 116 °

➔ φ = 87.5°

M. Collignon

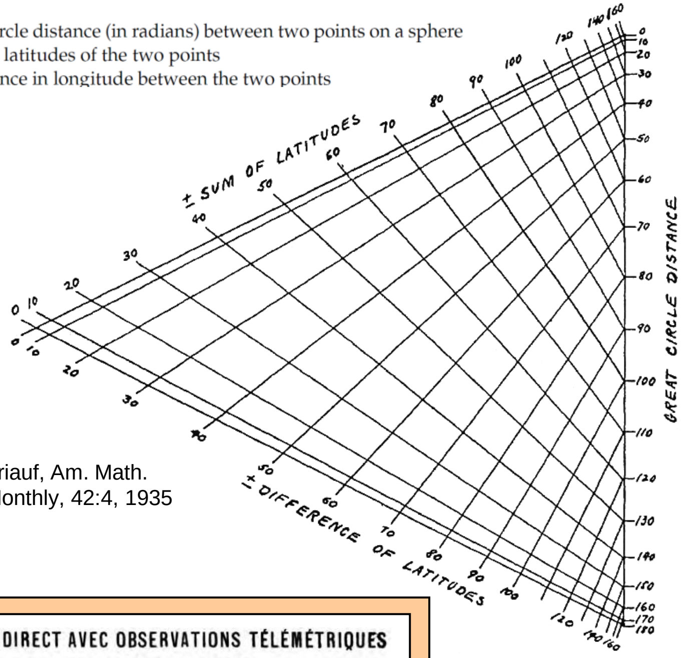
$$\begin{array}{c} \begin{array}{cccc} [y] & [x] & [z] & [t] \end{array} \\ \left| \begin{array}{cccc} \cos(\lambda-\lambda') & 1 & 0 & 1 \\ -\cos(\lambda+\lambda') & -1 & 0 & 1 \\ 2\cos\varphi & 0 & d & 1 \\ 0 & 2\cos L & -d & 1 \end{array} \right| = 0 \end{array}$$

Here the φ-scale lies the same distance *d* above the paper as the L-scale lies below it, but they are flattened to the paper. First, points A and B are joined by a line. Then for a given L (point C), all four points will be coplanar if the point on the flattened φ-scale is the same distance from AB as C and on a line parallel to AB. A transparent overlay of parallel lines is used to find φ.

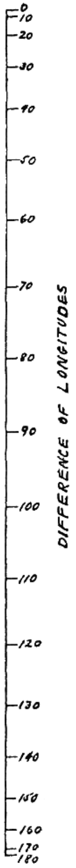
Great Circle Distance

$$\cos c = \cos a \cos b \cos L + \sin a \sin b$$

c is the great circle distance (in radians) between two points on a sphere
a and b are the latitudes of the two points
L is the difference in longitude between the two points

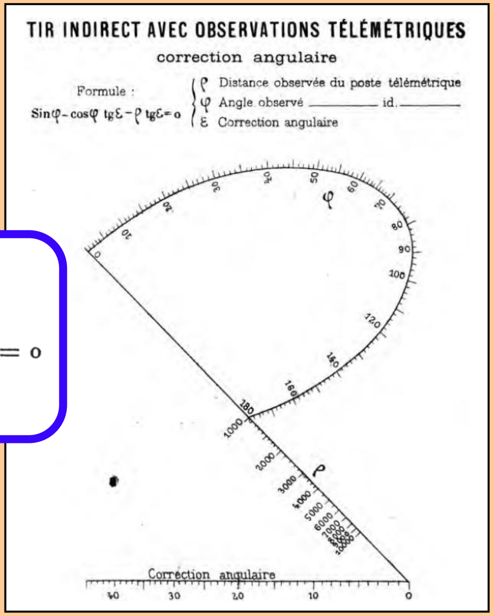


Friauf, Am. Math. Monthly, 42:4, 1935



$$\left| \begin{array}{ccc} 0 & 7\cos c & 1 \\ 6 & 10\cos L & 1 \\ \frac{84(\cos\alpha+\cos\beta)}{14(\cos\alpha+\cos\beta)-40} & \frac{140(\cos\alpha-\cos\beta)}{14(\cos\alpha+\cos\beta)-40} & 1 \end{array} \right| = 0$$

where $\alpha = a + b$ and $\beta = a - b$



Angular correction for land surveys

$$\sin\varphi - \cos\varphi \tan\varepsilon - \rho \tan\varepsilon = 0$$

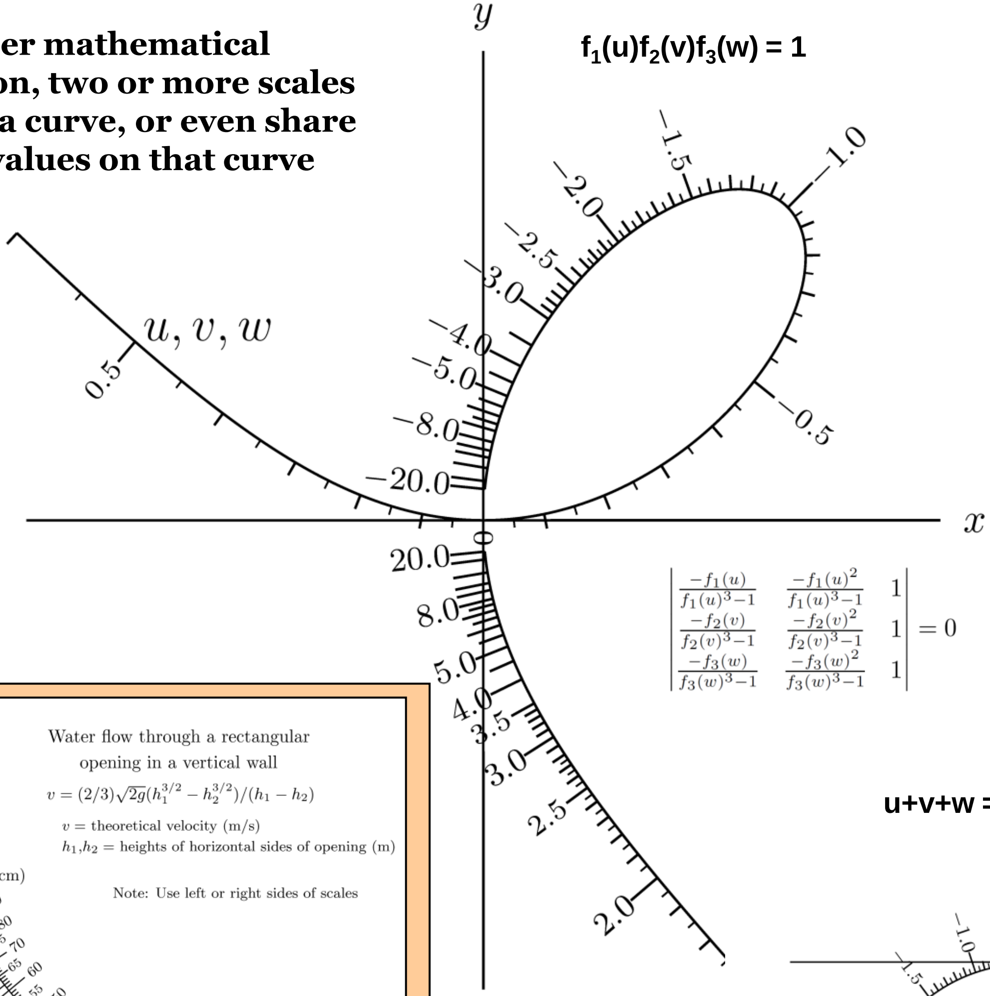
OCTOBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
September 2010 S M T W T F S 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30	November 2010 S M T W T F S 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30				1	2
3	4	5	6	● 7	8	9
10	11 **Columbus Day (Generally)**	12	13	14	15	16
17	18	19	20	21	○ 22	23
24	25	26	27	28	29	30
31 **Halloween**						

Shared-Scale Nomograms

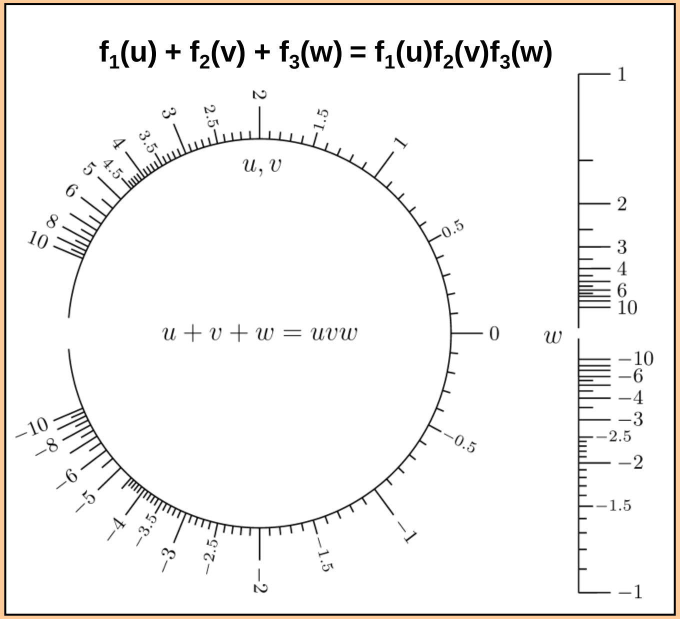
With proper mathematical preparation, two or more scales can share a curve, or even share the same values on that curve

$f_1(u)f_2(v)f_3(w) = 1$

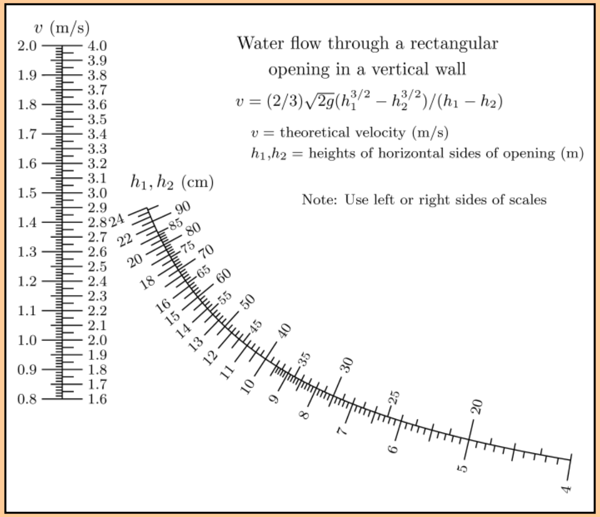


$$\begin{vmatrix} \frac{-f_1(u)}{f_1(u)^3-1} & \frac{-f_1(u)^2}{f_1(u)^3-1} & 1 \\ \frac{-f_2(v)}{f_2(v)^3-1} & \frac{-f_2(v)^2}{f_2(v)^3-1} & 1 \\ \frac{-f_3(w)}{f_3(w)^3-1} & \frac{-f_3(w)^2}{f_3(w)^3-1} & 1 \end{vmatrix} = 0$$

$f_1(u) + f_2(v) + f_3(w) = f_1(u)f_2(v)f_3(w)$



$$\begin{vmatrix} \frac{2}{f_1(u)^2+4} & \frac{f_1(u)}{f_1(u)^2+4} & 1 \\ \frac{2}{f_2(v)^2+4} & \frac{f_2(v)}{f_2(v)^2+4} & 1 \\ \frac{2}{3} & \frac{1}{3f_3(w)} & 1 \end{vmatrix} = 0$$



Water flow through a rectangular opening in a vertical wall

$v = (2/3)\sqrt{2g}(h_1^{3/2} - h_2^{3/2})/(h_1 - h_2)$

v = theoretical velocity (m/s)

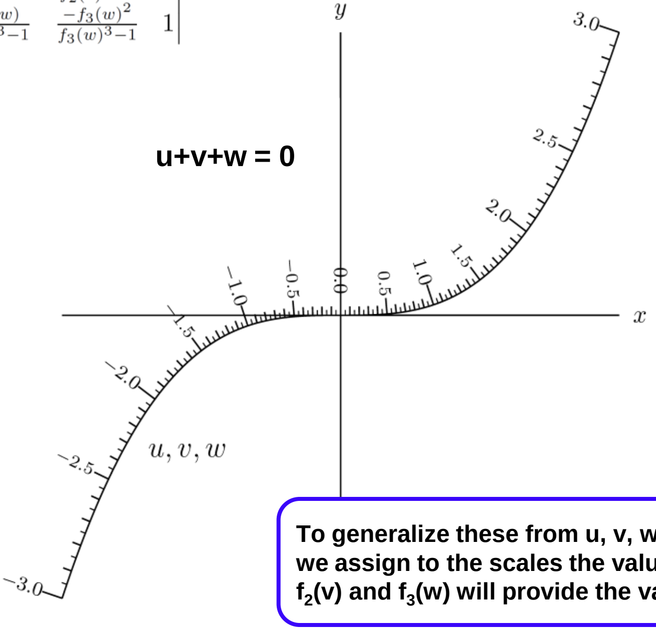
h_1,h_2 = heights of horizontal sides of opening (m)

Note: Use left or right sides of scales

Here the h_1 and h_2 scales are identical, and two ranges are marked on different sides of the scales.

$$\begin{vmatrix} 0 & 0.338v & 1 \\ \frac{1}{h_1} & h_1^{1/2} & 1 \\ \frac{1}{h_2} & h_2^{1/2} & 1 \end{vmatrix} = 0$$

u+v+w = 0



The 3 real roots of a cubic equation $ax^3+bx^2+cx+d = 0$ sum to $-b/a$, so a plot of x^3 marked with its x-values provides a single scale nomogram for addition.

To generalize these from u, v, w to $f_1(u)$, $f_2(v)$, $f_3(w)$, we assign to the scales the values for which $f_1(u)$, $f_2(v)$ and $f_3(w)$ will provide the values we see here.

NOVEMBER

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	1	2 Election Day	3	4	5	● 6
7	8	9	10	11 Veteran’s Day	12	13
14	15	16	17	18	19	20
○ 21	22	23	24	25 Thanksgiving	26	27
28	29	30				

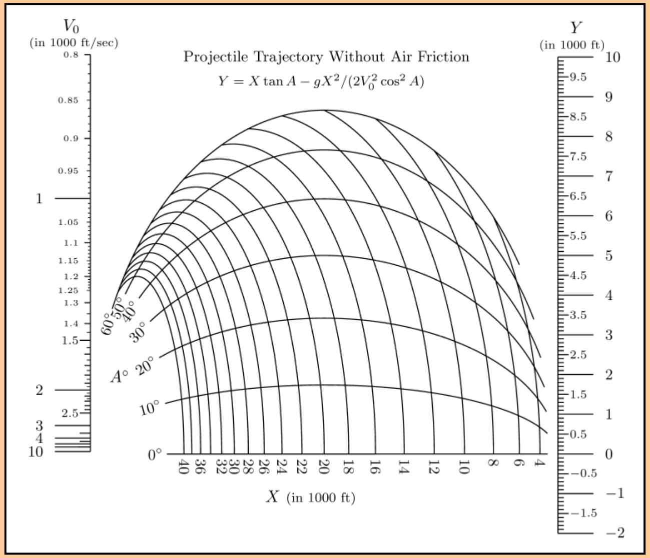
October 2010

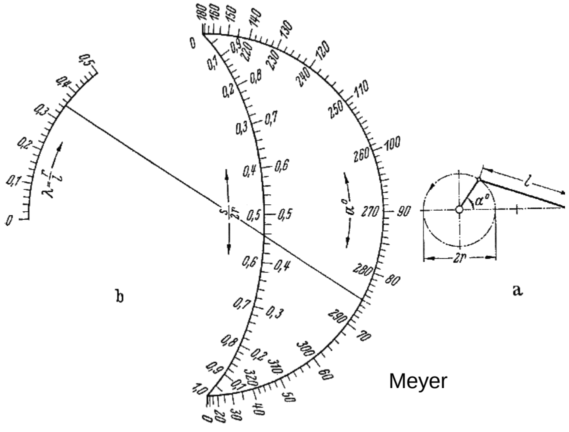
S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

December 2010

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

An Assortment of Nomograms

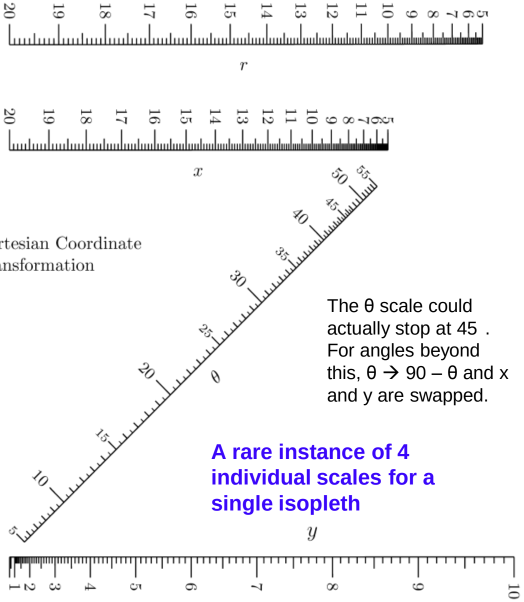




Meyer

$$s = r(1 - \cos\alpha) + l(1 - \sqrt{1 - \lambda^2 \sin^2\alpha}) \quad \text{with} \quad \lambda = \frac{r}{l}$$

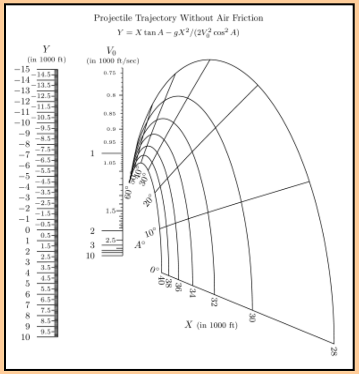
Polar-Cartesian Coordinate Transformation



The θ scale could actually stop at 45 . For angles beyond this, θ → 90 – θ and x and y are swapped.

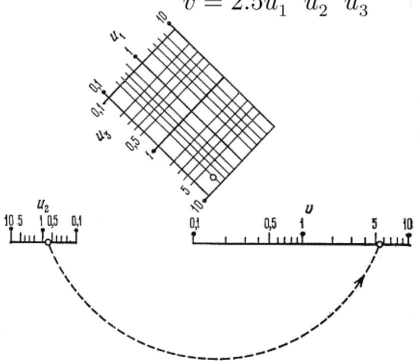
A rare instance of 4 individual scales for a single isopleth

Hoelscher

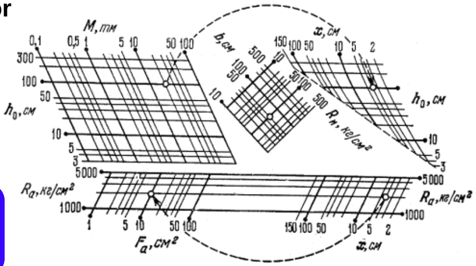


A projection transformation for greater accuracy at large X

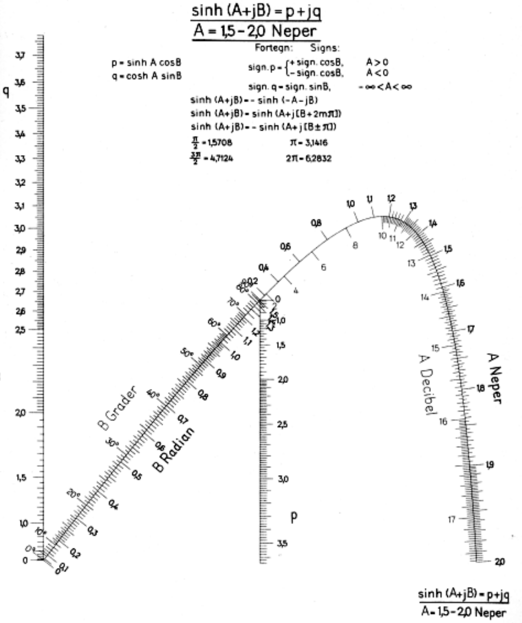
$$v = 2.5u_1^{0.6}u_2^{0.3}u_3^{0.9}$$



A compass is used here instead of a straightedge



$$bR_M = \frac{M}{x(h_0 - 0.5x)} = \frac{F_aR_a}{x}$$



sinh(A + jB) = p + jq

Rybner

Nomograms existed for a variety of vector and complex number calculations

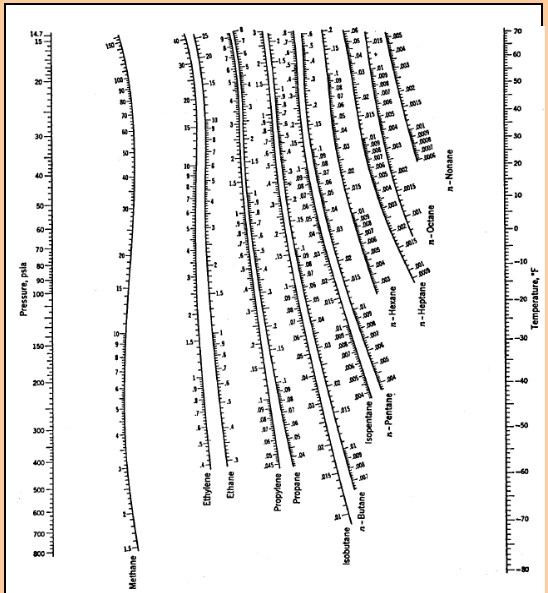


Fig. B-3a. Distribution coefficients ($K = y/x$) in light hydrocarbon systems, low-temperature range. [*C. L. Depriester, Chemical Eng. Progr. Symposium Ser.*, **49**(7), 1 (1953).]

DECEMBER

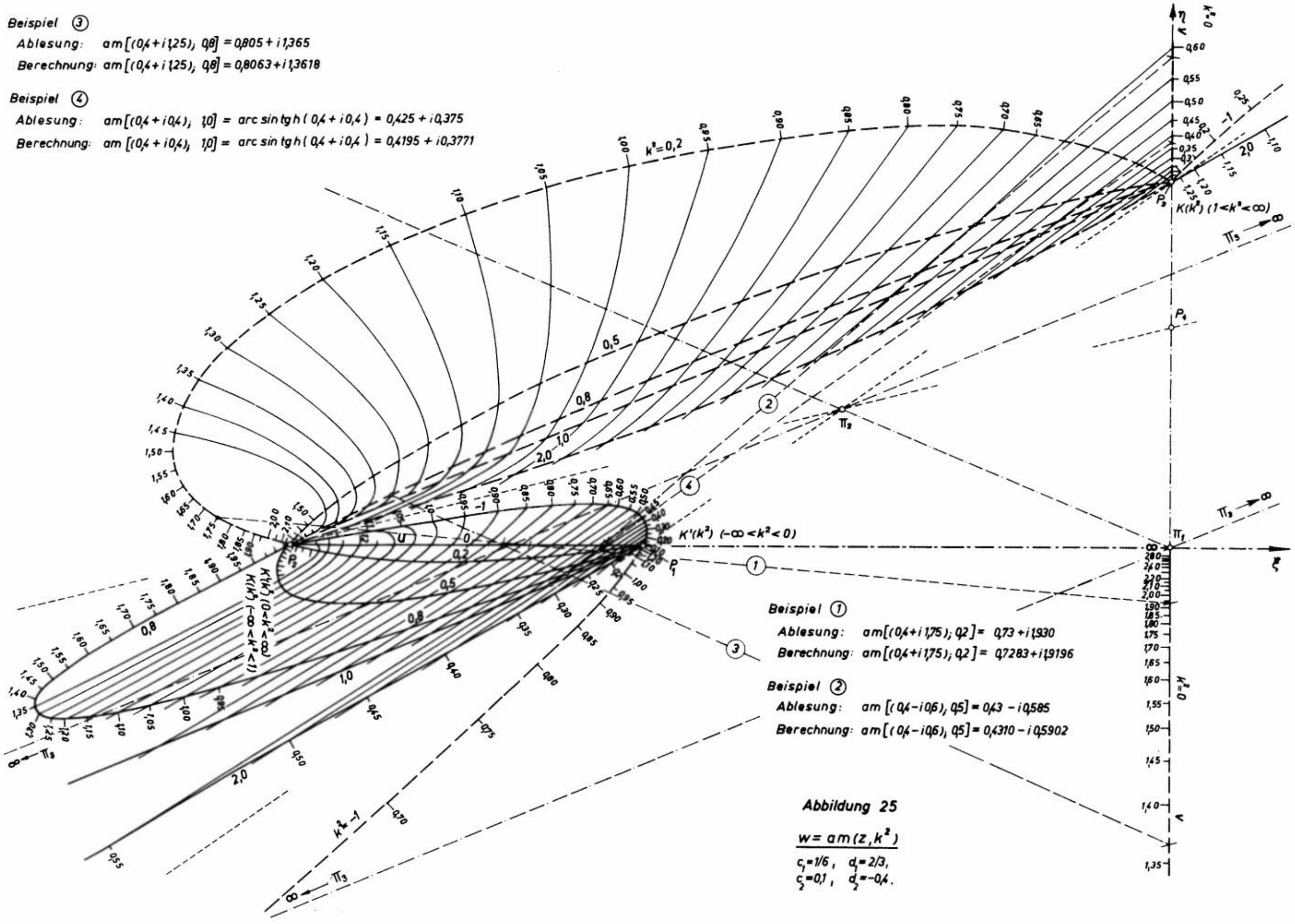
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
November 2010 S M T W T F S 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30	January 2011 S M T W T F S 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31		1	2	3	4
● 5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	○ 21	22	23	24 **Christmas Eve**	25 **Christmas Day**
26	27	28	29	30	31 **New Year’s Eve**	

A 2010 Calendar of Graphical Computers

THE AGE OF GRAPHICAL COMPUTING A.D. 1844-1974

Graphical Computers are fascinating artifacts in the history of mathematics. They possess an intrinsic charm well beyond their practical use.

- ➢ As a calculating aid graphical computers can solve very complicated formulas with amazing ease.
- ➢ As a curiosity graphical computers manifest the beauty of mathematics in a highly visual, highly creative way.



Most of the nomograms herein were created with the PyNomo software package of Leif Roschier found at http://www.pynomo.org. The calendar pages are based on an InDesign template created by Juliana Halvorson at http://www.graphmaster.com/calendarinstructions/. All other content ©2010 Ron Doerfler
Contact: rondoerfler@myreckonings.com